Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME								
CENTRE NUMBER						CANDIDATE NUMBER		

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (RW/CB) 311419/2

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1 **(a)** Find the Maclaurin series for $\sin^{-1}x$ up to and including the term in x^3. [5]

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(b) Deduce an approximation to $\int_0^{\frac{1}{5}} \frac{1}{\sqrt{1-u^2}}\,du$, giving your answer as a fraction. [1]

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2 The variables x and y are related by the differential equation

$$6\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x = t^2 + 10t + 13.$$

(a) Find the general solution for x in terms of t. [6]

(b) State an approximate solution for large positive values of t. [1]

3 By considering the binomial expansions of $\left(z+\frac{1}{z}\right)^4$ and $\left(z-\frac{1}{z}\right)^4$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cot^4\theta = \frac{\cos 4\theta + a\cos 2\theta + b}{\cos 4\theta - a\cos 2\theta + b},$$

where a and b are integers to be determined. [7]

4 The curve C has equation

$$4y^3 + (x+y)^6 = 109.$$

(a) Show that, at the point $(-4, 3)$ on C, $\frac{dy}{dx} = \frac{1}{17}$. [3]

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-4, 3)$. [5]

5 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

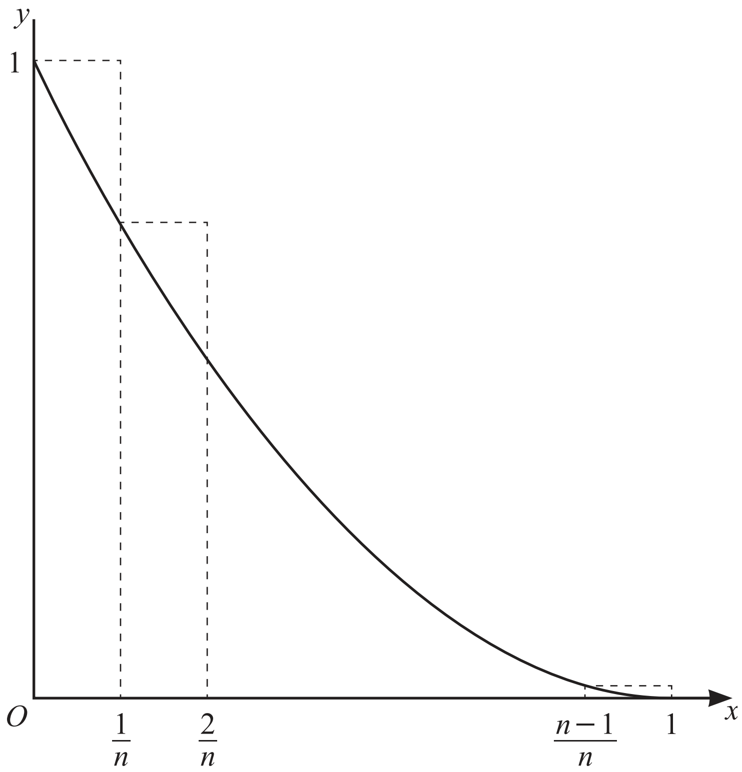
$$2\cosh^2 x = \cosh 2x + 1.$$ [3]

(b) Find the solution of the differential equation

$$\frac{dy}{dx} + 2y\tanh x = 1$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. [8]

6



The diagram shows the curve with equation $y = (1-x)^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (1-x)^2 \mathrm{d}x < U_n$, where

$$U_n = \frac{2n^2 + 3n + 1}{6n^2}.$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 (1-x)^2 \mathrm{d}x$. [4]

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(c) Show that $\lim\limits_{n\to\infty}(U_n - L_n) = 0$. [2]

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7 The integral I_n, where n is an integer, is defined by $I_n = \int_0^{\frac{4}{3}} (1+x^2)^{\frac{1}{2}n}\,dx$.

(a) Find the exact value of I_{-1} giving your answer in the form $\ln a$, where a is an integer to be determined. [2]

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(b) By considering $\frac{d}{dx}\left(x(1+x^2)^{\frac{1}{2}n}\right)$, or otherwise, show that

$$(n+1)I_n = nI_{n-2} + \tfrac{4}{3}\left(\tfrac{5}{3}\right)^n.$$ [5]

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(c) A curve has equation $y = x^2$, for $0 \leqslant x \leqslant \frac{2}{3}$. The arc length of the curve is denoted by s.

Use the substitution $u = 2x$ to show that $s = \frac{1}{2}I_1$ and find the exact value of s. [4]

8 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} a & -6a & 2a+2 \\ 0 & 1-a & 0 \\ 0 & 2-a & -1 \end{pmatrix}$$

where a is a constant with $a \neq 0$ and $a \neq 1$.

(a) Show that the equation $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ has a unique solution and interpret this situation geometrically. [3]

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(b) Show that the eigenvalues of **A** are a, $1-a$ and -1. [2]

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(c) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^4 = \mathbf{PDP}^{-1}$. [6]

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(d) Use the characteristic equation of **A** to find $\mathbf{A}^4$ in terms of **A** and a. [3]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.